Heavy Quark Expansion in $1/\hat{m}_Q$ and $|V_{cb}|$ Extraction

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Abstract

The dressed heavy quark mass $\hat{m}_Q = m_Q + \bar{\Lambda}$ with $\bar{\Lambda}$ being the binding energy is introduced to characterize the heavy hadrons containing a single heavy quark. A heavy quark expansion in terms of the inverse of the dressed heavy quark mass $1/\hat{m}_Q$ is presented with a complete decomposition of the full field and integrating out the small components. The heavy quark-antiquark coupling effects are included in the finite mass corrections. It is shown that the $1/\hat{m}_Q$ expansion is more favorable in application. The extraction of $|V_{cb}|$ from exclusive B decays is studied by using such a new expansion approach.

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I. INTRODUCTION

For a heavy hadron which contains a single heavy quark (bottom or charm), the heavy quark mass m_Q is much larger than the QCD energy scale Λ_{QCD} which characterizes the light degrees of freedom in the heavy hadron. The four momentum of the heavy quark can be expressed as $p^\mu = m_Q v^\mu + k^\mu$, where v^μ is taken to be the velocity of hadron at the rest frame, and k^μ is the residual momentum of the order of binding energy, which is much smaller than m_Q. The heavy quark symmetry [1–3] and its breaking effects are of particular importance in studying such hadrons. Consequently, the heavy quark effective theory (HQET) has been developed, where the effective Lagrangian is expanded in $1/m_Q$. In deriving the Lagrangian of HQET, the quark and antiquark are assumed to be conserved separately. Namely the heavy antiquarks are regarded as completely decoupled from the heavy quarks at the beginning. The transition matrix elements can also be represented in series of $1/m_Q$ through the heavy quark expansion (HQE) and evaluated order by order. HQET and HQE have been discussed by many authors [4–15]. In the past two decades they are widely used in studying heavy hadrons.

When the momentum of the heavy quark is much lower than the quark-antiquark pair creation threshold, an alternative framework of effective field theory for heavy quarks can directly be derived from the full QCD [16–19]. Just like for other effective theories, the basic idea is that some degrees of freedom characterizing higher scale physics can be decomposed and integrated out when we consider physics at low energy scales. Concretely speaking, for heavy quarks with $|\mathbf{p}| \ll 2m_Q$, one may perform a complete decomposition of the QCD full field into quark field and antiquark field via positive and negative energy components (see below) of a full field, and integrate out the small components of quark field and antiquark field, which leads to the so-called $1/m_Q$ corrections. When considering heavy quark (or antiquark) systems, one should further integrate in the contributions of heavy antiquark (or quark) components. As a consequence, additional $1/m_Q$ corrections arise from the quark-antiquark coupling terms in the full QCD. It should be noted that such a framework is distinguishable from the usual HQET in which the particle and antiparticle were assumed to be conserved separately and treated in a different way. For convenience, we refer to such a framework as a heavy quark effective field theory (HQEFT). Though the heavy quark-antiquark coupling effects vanish in the heavy quark limit, they are actually nonzero when one considers the finite mass contributions. It is then not surprising that the $1/m_Q$ corrections evaluated in HQEFT and HQET could be different though they are the same in the infinity mass limit. For instance, the transition matrix elements of $1/m_Q$ corrections concern less independent wave functions in HQEFT than in the usual HQET and the $1/m_Q$ order corrections at zero recoil automatically vanish in HQEFT, and there exist some relations between wave functions and heavy hadron masses in HQEFT.

HQEFT has been applied to explore various processes of heavy hadrons. In particular, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}|$ and $|V_{ub}|$ are extracted from both inclusive [20–23] and exclusive [17, 24–29] B decays. In the treatment of inclusive B decays, the dressed heavy quark mass $\hat{m}_Q = m_Q + \bar{\Lambda}$ as a whole enters the formulation, which implies that a “dressed heavy quark”-hadron duality is more reasonable than the naive heavy quark-hadron duality. As a consequence, when the inclusive decay rates of heavy hadrons are expressed in terms of the physical hadron masses, they receive no $1/\hat{m}_Q$ order corrections. This treatment not only suppresses the next-to-leading order contributions and makes the operator product expansion (OPE) reliable, but also diminishes the large

uncertainties arising from the heavy quark mass. For exclusive decays, HQEFT has also been demonstrated to be reliable. Whereas in our previous works the heavy quark expansion for the effective Lagrangian and transition matrix elements is carried out in powers of $1/m_Q$.

In this paper, we briefly review the description of HQEFT and show that a $1/\hat{m}_Q$ expansion is also consistently applicable to the heavy quark effective Lagrangian and transition matrix elements. In Sec. II, we first outline the derivation of a complete HQEFT and then extend it to the formulation in terms of $1/\hat{m}_Q$ expansion. In Sec. III, we present new formulae for HQE of heavy-to-heavy transition matrix elements by applying the $1/\hat{m}_Q$ expansion. In Sec. IV, we extract the CKM matrix element $|V_{cb}|$ based on the new formulation and the most recent experimental data. Our conclusions are given in Sec. V.

II. HEAVY QUARK EXPANSION IN TERMS OF $1/\hat{m}_Q$

The Lagrangian in the full QCD is

$$\mathcal{L}_{QCD} = \bar{Q}(i\not{D} - m_Q)Q + \mathcal{L}_{light}. \tag{1}$$

Q is the full field for heavy quark, and $\mathcal{L}_{light}$ represents the section containing no heavy quarks. Based on the principle of superposition, the field Q in quantum field theory is actually the superposition of two parts which correspond to the positive and negative energy components. Namely the full field Q can always be decomposed formally into positive and negative energy parts

$$Q = Q^+ + Q^-, \tag{2}$$

where Q^+ and Q^- may be expressed explicitly in the energy-momentum space as

$$\begin{aligned} Q^+(x) &= \int \frac{d^4p}{(2\pi)^4}\theta(p^0)Q^+(p)e^{-ip\cdot x}, && (3)\\ Q^-(x) &= \int \frac{d^4p}{(2\pi)^4}\theta(p^0)Q^-(p)e^{ip\cdot x} = \int \frac{d^4p}{(2\pi)^4}\theta(-p^0)Q^-(-p)e^{-ip\cdot x}. && (4) \end{aligned}$$

It is clear that Q^+ and Q^- correspond to the positive and negative energy parts of the full field Q, which are the so-called quark field and antiquark field, respectively. In the case for free quark fields, they can be expanded in terms of plane waves as

$$\begin{aligned} Q^+(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{m}{E}\sum_s b_s(p)u_s(p)e^{-ip\cdot x} \\ &= \int \frac{d^4p}{(2\pi)^4}(2\pi)2m\delta(p^2-m^2)\theta(p^0)\sum_s b_s(p)u_s(p)e^{-ip\cdot x}, && (5)\\ Q^-(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{m}{E}\sum_s d_s^\dagger(p)v_s(p)e^{ip\cdot x} \\ &= \int \frac{d^4p}{(2\pi)^4}(2\pi)2m\delta(p^2-m^2)\theta(p^0)\sum_s d_s^\dagger(p)v_s(p)e^{ip\cdot x}, && (6) \end{aligned}$$

where s is the spin index, b_s and $d_s^\dagger$ are the annihilation and creation operators respectively, and u_s and v_s are four-component spinors. In Dirac representation, they can be explicitly written as

$$u_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ \frac{\sigma\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \varphi_s, \tag{7}$$

$$v_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{p^0+m} \\ 1 \end{pmatrix} \chi_s \tag{8}$$

with φ_s being the two component Pauli spinor field that annihilates a heavy quark, and χ_s being the Pauli spinor field that creates a heavy antiquark. Later on we will see that the effective heavy quark and antiquark fields at $|\mathbf{p}| \ll 2m_Q$ exactly come from the "large" components of Q^+ and Q^-.

The generating functional in the full theory can be represented as

$$Z[j] = \int \mathcal{D}q\mathcal{D}Q e^{i\int d^4x(\mathcal{L}_{QCD}[q,Q]+j\phi)}. \tag{9}$$

In Eq.(9) and all the following relevant equations, we simply denote the source terms as $j\phi$ for convenience.

Introducing a vector v^μ with $v^2=1$, one can define the projecting operators

$$P_\pm = \frac{1\pm \not{v}}{2},$$

which satisfies

$$P_\pm^2 = P_\pm.$$

Then $Q^\pm$ can be written as

$$Q^+ = \Big(\frac{1+\not{v}}{2} + \frac{1-\not{v}}{2}\Big)Q^+ = \hat{Q}_v^+ + R_v^+, \tag{10}$$

$$Q^- = \Big(\frac{1-\not{v}}{2} + \frac{1+\not{v}}{2}\Big)Q^- = \hat{Q}_v^- + R_v^- \tag{11}$$

with

$$\hat{Q}_v^\pm \equiv \frac{1\pm\not{v}}{2}Q^\pm, \qquad R_v^\pm \equiv \frac{1\mp\not{v}}{2}Q^\pm. \tag{12}$$

At $|\mathbf{p}| \ll 2m_Q$, the field components R_v^+ and R_v^- become "small components" of quarks and antiquarks, while $\hat{Q}_v^+$ and $\hat{Q}_v^-$ are the "large components" [18, 19]. To be more explicit, taking $v=(1,0,0,0)$, one then has in the momentum space

$$\hat{Q}_v^+ \to \frac{1+\not{v}}{2}u_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi_s, \tag{13}$$

$$R_v^+ \to \frac{1-\not{v}}{2}u_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 0 \\ \frac{\sigma\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \varphi_s, \tag{14}$$

$$R_v^- \to \frac{1+\not{v}}{2}v_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{p^0+m} \\ 0 \end{pmatrix} \chi_s, \tag{15}$$

$$\hat{Q}_v^- \to \frac{1-\not{v}}{2}v_s(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \chi_s. \tag{16}$$

In this case one can decompose the full field Q as Eqs.(2), (10) and (11), and write the generating functional as

$$Z[j]=\int \mathcal{D}q\mathcal{D}\hat{Q}_v^{+}\mathcal{D}\hat{Q}_v^{-}\mathcal{D}R_v^{+}\mathcal{D}R_v^{-}e^{i\int d^4x(\mathcal{L}_{QCD}[q,\hat{Q}_v^{+},\hat{Q}_v^{-},R_v^{+},R_v^{-}]+j\phi)}. \tag{17}$$

Then one may integrate out the small components R_v^+ and R_v^- to get

$$Z[j]=\int \mathcal{D}q\mathcal{D}\hat{Q}_v^{+}\mathcal{D}\hat{Q}_v^{-}e^{i\int d^4x(\mathcal{L}_{light}+\hat{\mathcal{L}}_{Q,v}[\hat{Q}_v^{+},\hat{Q}_v^{-}]+j\phi)}. \tag{18}$$

$\hat{\mathcal{L}}_{Q,v}$ is the resulting Lagrangian for heavy section with the small components integrated out. It can also be derived equivalently by using the relevant Dirac equation of motion

$$(i\not{D}_{\parallel}-m_Q)R_v^{\pm}+i\not{D}_{\perp}\hat{Q}_v^{\pm}=0, \tag{19}$$

$$\bar{R}_v^{\pm}(-i\overleftarrow{\not{D}_{\parallel}}-m_Q)-\bar{\hat{Q}}_v^{\pm}i\overleftarrow{\not{D}_{\perp}}=0, \tag{20}$$

where $\not{D}_{\parallel}$, $\not{D}_{\perp}$, $\overleftarrow{\not{D}_{\parallel}}$, $\overleftarrow{\not{D}_{\perp}}$ and $\overleftarrow{D}^{\mu}$ are defined as

$$\begin{aligned}
&\not{D}_{\parallel}=\not{v}v\cdot D, && \not{D}_{\perp}=\not{D}-\not{v}v\cdot D, && \overleftarrow{\not{D}_{\parallel}}=\not{v}v\cdot\overleftarrow{D},\\
&\overleftarrow{\not{D}_{\perp}}=\overleftarrow{\not{D}}-\not{v}v\cdot\overleftarrow{D}, && \int\kappa\overleftarrow{D}^{\mu}\varphi=-\int\kappa D^{\mu}\varphi. &&
\end{aligned} \tag{21}$$

Clearly one has

$$\{\not{v},\not{D}_{\perp}\}=[\not{v},\not{D}_{\parallel}]=0. \tag{22}$$

$\hat{\mathcal{L}}_{Q,v}$ is found to be [17, 19]

$$\hat{\mathcal{L}}_{Q,v}=\hat{\mathcal{L}}_{Q,v}^{(++)}+\hat{\mathcal{L}}_{Q,v}^{(--)}+\hat{\mathcal{L}}_{Q,v}^{(+-)}+\hat{\mathcal{L}}_{Q,v}^{(-+)} \tag{23}$$

with

$$\begin{aligned}
\hat{\mathcal{L}}_{Q,v}^{(\pm\pm)} &= \bar{\hat{Q}}_v^{\pm}[i\hat{\not{D}}_v-m_Q]\hat{Q}_v^{\pm},\\
\hat{\mathcal{L}}_{Q,v}^{(\pm\mp)} &= \frac{1}{2m_Q}\bar{\hat{Q}}_v^{\pm}(-i\overleftarrow{\hat{\not{D}}}_v-m_Q)\Big(1-\frac{i\not{D}_{\parallel}+m_Q}{2m_Q}\Big)^{-1}(i\not{D}_{\perp})\hat{Q}_v^{\mp}\\
&= \frac{1}{2m_Q}\bar{\hat{Q}}_v^{\pm}(-i\overleftarrow{\not{D}_{\perp}})\Big(1-\frac{-i\overleftarrow{\not{D}_{\parallel}}+m_Q}{2m_Q}\Big)^{-1}(i\hat{\not{D}}_v-m_Q)\hat{Q}_v^{\mp},
\end{aligned} \tag{24}$$

where $i\hat{\not{D}}_v$ is defined as

$$i\hat{\not{D}}_v=i\not{D}_{\parallel}+\frac{1}{2m_Q}i\not{D}_{\perp}\Big(1-\frac{i\not{D}_{\parallel}+m_Q}{2m_Q}\Big)^{-1}i\not{D}_{\perp}, \tag{25}$$

and the operator $-i\overleftarrow{\hat{\not{D}}}_v$ can be obtained from $i\hat{\not{D}}_v$ by replacing D^{μ} with $-\overleftarrow{D}^{\mu}$.

To get a reliable expansion at low energies, the large momentum carried by the heavy quark should be removed. Generally this can be achieved by introducing new field variables

$$Q_v=e^{i\not{v}\hat{m}_Q v\cdot x}\hat{Q}_v,\qquad \bar{Q}_v=\bar{\hat{Q}}_v e^{-i\not{v}\hat{m}_Q v\cdot x}. \tag{26}$$

$\hat{m}_Q$ is a parameter with mass dimension. It can be chosen appropriately according to the physical picture for the process studied. In general a heavy quark within a hadron cannot truly be on shell due to strong interaction among heavy and light quarks as well as soft gluons. Thus one may write the total momentum of the heavy quark in a hadron as $P_Q = m_Q v + k = \hat{m}_Q v + \tilde{k}$, where $v \cdot \tilde{k}$ is the part which depends on heavy flavor and is suppressed by the heavy quark mass; $\hat{m}_Q$ is defined as the sum of the heavy quark mass and the binding energy $\bar{\Lambda}$ that reflects the nonperturbative effects of strong interaction and relates to the light constituents in the heavy hadron, $\hat{m}_Q = m_Q + \bar{\Lambda}$. In such a consideration, the residual momentum $k = \bar{\Lambda} v + \tilde{k}$ as a whole characterizes the off-shellness of the heavy quark in the heavy hadron. Namely, the total residual momentum $k = \bar{\Lambda} v + \tilde{k}$ of the heavy quark is assumed to comprise the main contributions of the light degrees of freedom in the heavy hadron containing a single heavy quark. In such a physical picture, the heavy quark may be regarded as a "dressed heavy quark", and the heavy hadron containing a single heavy quark is more reliable to be considered as a dualized particle of a "dressed heavy quark". Different from the "heavy quark"-hadron duality in the usual heavy quark effective theory, what we are considering is the physical picture of the "dressed heavy quark"-hadron duality. As a consequence, the wave functions defined in the next section should have a weaker dependence on the light constituents of heavy hadrons.

With the definition (26), the Lagrangian (23) can be written in terms of Q_v^+ and $\bar{Q}_v^+$, which carry only the small residual momentum $\tilde{k}^\mu = p_H^\mu - \hat{m}_Q v^\mu$. Explicitly, one has[19]

$$
\begin{aligned}
\mathcal{L}_{Q,v} &= \mathcal{L}_{Q,v}^{I} + \mathcal{L}_{Q,v}^{II} = \mathcal{L}_{Q,v}^{(0)} + \mathcal{L}_{Q,v}^{(1/\hat{m}_Q)}, && (27)\\
\mathcal{L}_{Q,v}^{I} &= \mathcal{L}_{Q,v}^{(++)} + \mathcal{L}_{Q,v}^{(--)} = \bar{Q}_v (i\not{\mathcal{D}}_v) Q_v \equiv \mathcal{L}_{Q,v}^{(0)} + \mathcal{L}_{Q,v}^{I(1/\hat{m}_Q)}, && (28)\\
\mathcal{L}_{Q,v}^{II} &= \mathcal{L}_{Q,v}^{(+-)} + \mathcal{L}_{Q,v}^{(-+)}\\
&= \frac{1}{2\hat{m}_Q} \bar{Q}_v (-i\overleftarrow{\not{\mathcal{D}}}_v) e^{2i\not{v}\hat{m}_Q v\cdot x} \Big(1 - \frac{i\not{D}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} (i\not{D}_\perp) Q_v\\
&= \frac{1}{2\hat{m}_Q} \bar{Q}_v (-i\overleftarrow{\not{D}}_\perp) \Big(1 - \frac{-i\overleftarrow{\not{D}}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} e^{-2i\not{v}\hat{m}_Q v\cdot x} (i\not{\mathcal{D}}_v) Q_v\\
&\equiv \mathcal{L}_{Q,v}^{II(1/\hat{m}_Q)} && (29)
\end{aligned}
$$

with

$$
\begin{aligned}
i\not{\mathcal{D}}_v &= i\not{D}_\parallel + \bar{\Lambda} + \frac{1}{2\hat{m}_Q} i\not{D}_\perp \Big(1 - \frac{i\not{D}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} i\not{D}_\perp,\\
-i\overleftarrow{\not{\mathcal{D}}}_v &= -i\overleftarrow{\not{D}}_\parallel + \bar{\Lambda} + \frac{1}{2\hat{m}_Q} (-i\overleftarrow{\not{D}}_\perp) \Big(1 - \frac{-i\overleftarrow{\not{D}}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} (-i\overleftarrow{\not{D}}_\perp). && (30)
\end{aligned}
$$

In Eq.(27) we use $\mathcal{L}_{Q,v}^{(0)}$ to denote the leading term in the $1/\hat{m}_Q$ expansion of the Lagrangian $\mathcal{L}_{Q,v}$, and $\mathcal{L}_{Q,v}^{(1/\hat{m}_Q)}$ contains all $1/\hat{m}_Q$ corrections to $\mathcal{L}_{Q,v}^{(0)}$. From Eqs.(27)-(29) one has

$$
\begin{aligned}
\mathcal{L}_{Q,v}^{(0)} &= \bar{Q}_v (i\not{D}_\parallel + \bar{\Lambda}) Q_v, && (31)\\
\mathcal{L}_{Q,v}^{I(1/\hat{m}_Q)} &= \bar{Q}_v \frac{1}{2\hat{m}_Q} i\not{D}_\perp \Big(1 - \frac{i\not{D}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} i\not{D}_\perp Q_v, && (32)\\
\mathcal{L}_{Q,v}^{(1/\hat{m}_Q)} &= \mathcal{L}_{Q,v}^{I(1/\hat{m}_Q)} + \mathcal{L}_{Q,v}^{II(1/\hat{m}_Q)}. && (33)
\end{aligned}
$$

Note that

$$Q_v = Q_v^+ + Q_v^-, \tag{34}$$

so the effective Lagrangian $\mathcal{L}_{Q,v}$ is complete for the large component of heavy quark and antiquark. In deriving $\mathcal{L}_{Q,v}$ we have only integrated over the small component ($R_v = R_v^+ + R_v^-$) of heavy quark and antiquark fields.

In the above discussions, the binding energy $\bar{\Lambda}$ is introduced based on physical consideration of the heavy-light systems. Formally it can be defined consistently via the normalization of hadron states as follows. The hadron state $|H\rangle$ in full theory is normalized as

$$\langle H(p)|\bar{Q}\gamma^\mu Q|H(p)\rangle = 2p_H^\mu = 2m_H v^\mu, \tag{35}$$

where $p_H^\mu = m_H v^\mu$ is the momentum of the heavy hadron H. In the effective theory at low energies, one may introduce an effective heavy hadron state $|H_v\rangle$, which is heavy flavor independent, and normalized as

$$\langle H_v|\bar{Q}_v\gamma^\mu Q_v|H_v\rangle = 2\bar{\Lambda}v^\mu. \tag{36}$$

It is then related to the heavy hadron state $|H\rangle$ via

$$\frac{1}{\sqrt{m_{H'}m_H}}\langle H'|\bar{Q}'\Gamma Q|H\rangle = \frac{1}{\sqrt{\bar{\Lambda}_{H'}\bar{\Lambda}_H}}\langle H'_{v'}|J_{Q,v}e^{i\int d^4x\mathcal{L}_{Q,v}^{(1/\hat{m}_Q)}}|H_v\rangle, \tag{37}$$

where Γ denotes Dirac matrixes, $\bar{\Lambda}_H = m_H - m_Q$ and $\bar{\Lambda}_{H'} = m_{H'} - m_{Q'}$ are the mass differences between heavy hadrons and heavy quarks, while

$$\bar{\Lambda} = \lim_{m_Q\to\infty} \bar{\Lambda}_H$$

is independent of the heavy flavor and reflects the contributions of light degrees of freedom in the hadron. $J_{Q,v}$ in Eq.(37) is derived from the current $\bar{Q}\Gamma Q$. It will be given explicitly in Sec.III.

The basic framework of HQEFT has been derived and discussed in the previous papers[16–19]. Here we reexpress the effective Lagrangian in terms of the expansion $1/\hat{m}_Q$ instead of $1/m_Q$. The binding energy arising from the "longitudinal residual momentum" of the heavy quark is absorbed into the heavy quark mass to be given as the dressed heavy quark mass $\hat{m}_Q = m_Q + \bar{\Lambda}$, so that the flavor independent nonperturbative contributions of the light degrees of freedom are effectively included in the dressed heavy quark. This treatment is consistent with the physical picture of a heavy kernel (the dressed heavy quark) surrounded with the clouds of light degrees of freedom which mainly reflect the small "transverse residual momentum" of heavy quark. As $\hat{m}_Q > m_Q$, an expansion in $1/\hat{m}_Q$ is expected to be more convergent and reliable, especially for charm quark systems. Actually, it has been shown in the inclusive decays[20, 21] that by using $\hat{m}_Q$ instead of adopting m_Q and $\bar{\Lambda}$ separately in the calculations, the results get less uncertainties and the order $1/\hat{m}_Q$ corrections are automatically absent. In this note, we will show that based on the new formulation of Lagrangian given in Eqs.(27)-(29), one can also consistently perform $1/\hat{m}_Q$ expansion for the exclusive decays.

III. TRANSITION MATRIX ELEMENTS IN $1/\hat{m}_Q$ EXPANSION

Similar to the derivation of the effective Lagrangian $\mathcal{L}_{Q,v}$, the heavy quark current $J(x) = \bar{Q}'(x)\Gamma Q(x)$ in full QCD can also be transformed into the following form by integrating out the small component R_v,

$$\begin{aligned} J(x) \to J_{Q,v}(x) &= \bar{\hat{Q}}'_{v'}(x)\Gamma\hat{Q}_v(x) + \bar{\hat{Q}}'_{v'}(x)\Gamma\hat{W}_v\hat{Q}_v(x) \\ &\quad + \bar{\hat{Q}}'_{v'}(x)\overleftarrow{\hat{W}}_{v'}\Gamma\hat{Q}_v(x) + \bar{\hat{Q}}'_{v'}(x)\overleftarrow{\hat{W}}_{v'}\Gamma\hat{W}_v\hat{Q}_v(x) \\ &= \bar{Q}'_{v'}(x)e^{i\not{v}'\hat{m}_{Q'}v'\cdot x}\Gamma e^{-i\not{v}\hat{m}_Q v\cdot x}Q_v(x) \\ &\quad + \bar{Q}'_{v'}(x)e^{i\not{v}'\hat{m}_{Q'}v'\cdot x}\Gamma e^{i\not{v}\hat{m}_Q v\cdot x}W_v Q_v(x) \\ &\quad + \bar{Q}'_{v'}(x)\overleftarrow{W}_{v'}e^{-i\not{v}'\hat{m}_{Q'}v'\cdot x}\Gamma e^{-i\not{v}\hat{m}_Q v\cdot x}Q_v(x) \\ &\quad + \bar{Q}'_{v'}(x)\overleftarrow{W}_{v'}e^{-i\not{v}'\hat{m}_{Q'}v'\cdot x}\Gamma e^{i\not{v}\hat{m}_Q v\cdot x}W_v Q_v(x), \end{aligned} \tag{38}$$

where

$$\begin{aligned} \hat{W}_v &= \frac{1}{2m_Q}\Big(1 - \frac{i\not{D}_\parallel + m_Q}{2m_Q}\Big)^{-1} i\not{D}_\perp, \\ W_v &= \frac{1}{2\hat{m}_Q}\Big(1 - \frac{i\not{D}_\parallel + \bar{\Lambda}}{2\hat{m}_Q}\Big)^{-1} i\not{D}_\perp, \\ \overleftarrow{\hat{W}}_{v'} &= \frac{1}{2m_{Q'}}(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\overleftarrow{\not{D}}_\parallel + m_{Q'}}{2m_{Q'}}\Big)^{-1}, \\ \overleftarrow{W}_{v'} &= \frac{1}{2\hat{m}_{Q'}}(-i\overleftarrow{\not{D}}_\perp)\Big(1 - \frac{-i\overleftarrow{\not{D}}_\parallel + \bar{\Lambda}'}{2\hat{m}_{Q'}}\Big)^{-1} \end{aligned} \tag{39}$$

with

$$\overleftarrow{\not{D}}_\parallel = \not{v}'v'\cdot\overleftarrow{D}, \qquad \overleftarrow{\not{D}}_\perp = \overleftarrow{\not{D}} - \not{v}'v'\cdot\overleftarrow{D}, \tag{40}$$

which have minor differences to the formulae in (21).

Using Eq.(22), and noting that

$$e^{iA\not{v}}\frac{1+\sigma\not{v}}{2} = e^{iA\sigma}\frac{1+\sigma\not{v}}{2}, \tag{41}$$

where $\sigma = \pm 1$ and A is a c-number, we write $J_{Q,v}$ as

$$\begin{aligned} J_{Q,v}(x) &= \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v' - \sigma\hat{m}_Q v)\cdot x}\bar{Q}'_{v'}(x)\frac{1+\sigma'\not{v}'}{2}(\Gamma + \Gamma W_v \\ &\quad + \overleftarrow{W}_{v'}\Gamma + \overleftarrow{W}_{v'}\Gamma W_v)\frac{1+\sigma\not{v}}{2}Q_v(x). \end{aligned} \tag{42}$$

Furthermore, both $J_{Q,v}$ and $\mathcal{L}_{Q,v}$ can be expanded in terms of $1/\hat{m}_{Q^{(\prime)}}$. Explicitly one obtains

$$
\begin{aligned}
J_{Q,v}(x) &= J^{(0)}_{Q,v}(x) + J^{(1/\hat{m}_Q)}_{Q,v}(x), && (43)\\
J^{(0)}_{Q,v}(x) &= \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v'-\sigma\hat{m}_Q v)\cdot x}\bar{Q}'_{v'}(x)\frac{1+\sigma'\ \not{v}'}{2}\Gamma\frac{1+\sigma\ \not{v}}{2}Q_v(x), && (44)\\
J^{(1/\hat{m}_Q)}_{Q,v}(x) &= \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v'-\sigma\hat{m}_Q v)\cdot x}\bar{Q}'_{v'}(x)\frac{1+\sigma'\ \not{v}'}{2}\Big[\frac{1}{2\hat{m}_Q}\Gamma i\not{D}_\perp + \frac{1}{2\hat{m}_{Q'}}(-i\overleftarrow{\not{D}_\perp})\Gamma \\
&\quad + \frac{1}{4\hat{m}_Q^2}\Gamma(i\not{D}_\parallel + \bar{\Lambda})i\not{D}_\perp + \frac{1}{4\hat{m}_{Q'}^2}(-i\overleftarrow{\not{D}_\perp})(-i\overleftarrow{\not{D}_\parallel} + \bar{\Lambda}')\Gamma \\
&\quad + \frac{1}{4\hat{m}_Q\hat{m}_{Q'}}(-i\overleftarrow{\not{D}_\perp})\Gamma(i\not{D}_\perp) + O\Big(\frac{1}{\hat{m}^3_{Q^{(\prime)}}}\Big)\Big]\frac{1+\sigma\ \not{v}}{2}Q_v(x) && (45)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{L}^{I(1/\hat{m}_Q)}_{Q,v} &= \sum_{\varepsilon=\pm 1}\bar{Q}_v\frac{1+\varepsilon\ \not{v}}{2}\Big[\frac{(i\not{D}_\perp)^2}{2\hat{m}_Q} + \frac{1}{4\hat{m}_Q^2}i\not{D}_\perp(i\not{D}_\parallel + \bar{\Lambda})i\not{D}_\perp \\
&\quad + O\Big(\frac{1}{\hat{m}_Q^3}\Big)\Big]\frac{1+\varepsilon\ \not{v}}{2}Q_v, && (46)\\
\mathcal{L}^{II(1/\hat{m}_Q)}_{Q,v} &= \sum_{\varepsilon=\pm 1} e^{2i\varepsilon\hat{m}_Q v\cdot x}\bar{Q}_v\frac{1+\varepsilon\ \not{v}}{2}\Big[\frac{1}{2\hat{m}_Q}(-i\overleftarrow{\not{D}_\parallel} + \bar{\Lambda})i\not{D}_\perp \\
&\quad + \frac{1}{4\hat{m}_Q^2}(-i\overleftarrow{\not{D}_\parallel} + \bar{\Lambda})(i\not{D}_\parallel + \bar{\Lambda})i\not{D}_\perp \\
&\quad + \frac{1}{4\hat{m}_Q^2}(-i\overleftarrow{\not{D}_\perp})^2 i\not{D}_\perp + O\Big(\frac{1}{\hat{m}_Q^3}\Big)\Big]\frac{1-\varepsilon\ \not{v}}{2}Q_v. && (47)
\end{aligned}
$$

Then the effective current in terms of $1/\hat{m}_Q$ expansion is obtained:

$$
\begin{aligned}
J^{eff}_{Q,v}(x) &\equiv \langle J_{Q,v}(x)e^{i\int d^4y\mathcal{L}^{(1/\hat{m}_Q)}_{Q,v}}\rangle \\
&= \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v'-\sigma\hat{m}_Q v)\cdot x}\bar{Q'}_{v'}\frac{1+\sigma'\ \not{v}'}{2}\Big[\Gamma - \frac{1}{2\hat{m}_Q}O_1(\Gamma) - \frac{1}{2\hat{m}_{Q'}}O_1'(\Gamma) \\
&\quad - \frac{1}{4\hat{m}_Q^2}O_2(\Gamma) - \frac{1}{4\hat{m}_{Q'}^2}O_2'(\Gamma) + \frac{1}{4\hat{m}_Q^2}O_3(\Gamma) + \frac{1}{4\hat{m}_{Q'}^2}O_3'(\Gamma) \\
&\quad + \frac{1}{4\hat{m}_{Q'}\hat{m}_Q}O_4(\Gamma) + O\Big(\frac{1}{\hat{m}^3_{Q^{(\prime)}}}\Big)\Big]\frac{1+\sigma\ \not{v}}{2}Q_v, && (48)
\end{aligned}
$$

where the operators are defined as

$$
\begin{aligned}
O_1(\Gamma) &= \Gamma\frac{1}{i\not{D}_\parallel+\bar{\Lambda}}(i\not{D}_\perp)^2,\\
O_1'(\Gamma) &= (-i\overleftarrow{\not{D}}_\perp)^2\frac{1}{-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}'}\Gamma,\\
O_2(\Gamma) &= \Gamma\frac{1}{i\not{D}_\parallel+\bar{\Lambda}}(i\not{D}_\perp)(i\not{D}_\parallel+\bar{\Lambda})i\not{D}_\perp,\\
O_2'(\Gamma) &= (-i\overleftarrow{\not{D}}_\perp)(-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}')(-i\overleftarrow{\not{D}}_\perp)\frac{1}{-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}'}\Gamma,\\
O_3(\Gamma) &= \Gamma\frac{1}{i\not{D}_\parallel+\bar{\Lambda}}(i\not{D}_\perp)^2\frac{1}{i\not{D}_\parallel+\bar{\Lambda}}(i\not{D}_\perp)^2,\\
O_3'(\Gamma) &= (-i\overleftarrow{\not{D}}_\perp)^2\frac{1}{-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}'}(-i\overleftarrow{\not{D}}_\perp)^2\frac{1}{-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}'}\Gamma,\\
O_4(\Gamma) &= (-i\overleftarrow{\not{D}}_\perp)^2\frac{1}{-i\overleftarrow{\not{D}}_\parallel+\bar{\Lambda}'}\Gamma\frac{1}{i\not{D}_\parallel+\bar{\Lambda}}(i\not{D}_\perp)^2.
\end{aligned}
\tag{49}
$$

Eq.(48) can be derived by using

$$
\frac{i}{i\not{D}_\parallel+\bar{\Lambda}}
\tag{50}
$$

as the propagator when Q_v and $\bar{Q}_v$ fields are contracted. The feasibility of this treatment is shown in the appendix.

Note that the effective current $J_{Q,v}^{eff}$ consists of Q^+ to Q^+ and Q^- to Q^- components as well as mixing ones of Q^+ to Q^- and Q^- to Q^+. When the effective field Q_v in $J_{Q,v}^{eff}$ acts on a specific hadron state, the state will pick up the proper component. That is, the hadron containing a single heavy quark (antiquark) picks up Q^+ (Q^-) and cancels Q^- (Q^+).

Now the heavy quark expansion for any heavy-to-heavy transition matrix elements can be represented as

$$
\begin{aligned}
\mathcal{A} &= \langle H'_{v'}|J_{Q,v}e^{i\int d^4x\mathcal{L}_{Q,v}^{(1/\hat{m}_Q)}}|H_v\rangle = \langle H'_{v'}|J_{Q,v}^{eff}|H_v\rangle\\
&= \langle H'_{v'}|\bar{Q}'_{v'}\Gamma Q_v|H_v\rangle - \frac{1}{2\hat{m}_Q}\langle H'_{v'}|\bar{Q}'_{v'}O_1(\Gamma)Q_v|H_v\rangle - \frac{1}{2\hat{m}_{Q'}}\langle H'_{v'}|\bar{Q}'_{v'}O_1'(\Gamma)Q_v|H_v\rangle\\
&\quad -\frac{1}{4\hat{m}_Q^2}\langle H'_{v'}|\bar{Q}'_{v'}O_2(\Gamma)Q_v|H_v\rangle - \frac{1}{4\hat{m}_{Q'}^2}\langle H'_{v'}|\bar{Q}'_{v'}O_2'(\Gamma)Q_v|H_v\rangle\\
&\quad +\frac{1}{4\hat{m}_Q^2}\langle H'_{v'}|\bar{Q}'_{v'}O_3(\Gamma)Q_v|H_v\rangle + \frac{1}{4\hat{m}_{Q'}^2}\langle H'_{v'}|\bar{Q}'_{v'}O_3'(\Gamma)Q_v|H_v\rangle\\
&\quad +\frac{1}{4\hat{m}_{Q'}\hat{m}_Q}\langle H'_{v'}|\bar{Q}'_{v'}O_4(\Gamma)Q_v|H_v\rangle + O\Big(\frac{1}{\hat{m}_{Q^{(\prime)}}^3}\Big).
\end{aligned}
\tag{51}
$$

When $v' = v$, one gets from Eqs.(35)-(37) and (51)

$$\bar{\Lambda}_H = \bar{\Lambda} - \frac{1}{2\hat{m}_Q}\langle H_v|\bar{Q}_v O_1(\not{v})Q_v|H_v\rangle - \frac{1}{4\hat{m}_Q^2}\langle H_v|\bar{Q}_v(O_2(\not{v}) - O_3(\not{v})Q_v|H_v\rangle + \frac{1}{8\hat{m}_Q^2}\langle H_v|\bar{Q}_v O_4(\not{v})Q_v|H_v\rangle + O\Big(\frac{1}{\hat{m}_Q^3}\Big). \tag{52}$$

It is seen from Eq.(52) that the heavy flavor dependence of $\bar{\Lambda}_H$ can be attributed to the heavy-to-heavy transition matrix elements, or relevant wave functions. The heavy hadron mass is then given by

$$m_H = m_Q + \bar{\Lambda}_H = m_Q + \bar{\Lambda} + O(1/\hat{m}_Q) = \hat{m}_Q\left(1 + O(1/\hat{m}_Q^2)\right), \tag{53}$$

which is the fact that the hadron mass consists of the dressed heavy quark mass and the residual mass suppressed by $1/\hat{m}_Q$.

To be concrete, we study the weak transition matrix elements between ground state pseudoscalar and vector mesons. They can be described by 18 form factors:

$$\begin{aligned}
&\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle = \sqrt{m_D m_B}[h_+(\omega)(v+v')^\mu + h_-(\omega)(v-v')^\mu],\\
&\langle D^*(v',\epsilon')|\bar{c}\gamma^\mu b|B(v)\rangle = i\sqrt{m_{D^*}m_B}h_V(\omega)\epsilon^{\mu\nu\alpha\beta}\epsilon'^*_\nu v'_\alpha v_\beta,\\
&\langle D^*(v',\epsilon')|\bar{c}\gamma^\mu\gamma^5 b|B(v)\rangle = \sqrt{m_{D^*}m_B}[h_{A_1}(\omega)(1+\omega)\epsilon'^{*\mu} - h_{A_2}(\omega)(\epsilon'^*\cdot v)v^\mu\\
&\qquad -h_{A_3}(\omega)(\epsilon'^*\cdot v)v'^\mu],\\
&\langle D^*(v',\epsilon')|\bar{c}\gamma^\mu b|B^*(v,\epsilon)\rangle = \sqrt{m_{D^*}m_{B^*}}\{-(\epsilon\cdot\epsilon'^*)[h_1(\omega)(v+v')^\mu + h_2(\omega)(v-v')^\mu]\\
&\qquad +h_3(\omega)(\epsilon'^*\cdot v)\epsilon^\mu + h_4(\omega)(\epsilon\cdot v')\epsilon'^{*\mu} - (\epsilon\cdot v')(\epsilon'^*\cdot v)[h_5(\omega)v^\mu + h_6(\omega)v'^\mu]\},\\
&\langle D^*(v',\epsilon')|\bar{c}\gamma^\mu\gamma^5 b|B^*(v,\epsilon)\rangle = i\sqrt{m_{D^*}m_{B^*}}\{\epsilon^{\mu\nu\alpha\beta}\{\epsilon_\alpha\epsilon'^*_\beta[h_7(\omega)(v+v')_\nu\\
&\qquad +h_8(\omega)(v-v')_\nu] + v'_\alpha v_\beta[h_9(\omega)(\epsilon'^*\cdot v)\epsilon_\nu + h_{10}(\omega)(\epsilon\cdot v')\epsilon'^*_\nu]\}\\
&\qquad +\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha\epsilon'^*_\beta v_\gamma v'_\delta[h_{11}(\omega)v^\mu + h_{12}(\omega)v'^\mu]\},
\end{aligned} \tag{54}$$

where $\epsilon^{(')\mu}$ is the polarization vector of the vector meson, and ω is the product of the four-velocities of heavy mesons, $\omega = v\cdot v'$.

On the other hand, for such transitions between $(Q^+\bar{q})$ states one can rewrite Eq.(51) as

$$\begin{aligned}
\mathcal{A} = \langle H'_{v'}|\bar{Q}'^+_{v'}\Big\{&\Gamma - \frac{1}{2\hat{m}_Q}\Gamma\frac{-P_+}{\bar{\Lambda}+iv\cdot D}\Big(D_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big) - \frac{1}{2\hat{m}_{Q'}}\Big(\overleftarrow{D}_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big)\\
&\frac{-P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma - \frac{1}{4\hat{m}_Q^2}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\Big[\Big(D_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big)(iv\cdot D - \bar{\Lambda}) - iv_\alpha D_\beta F^{\alpha\beta}\\
&+v_\alpha\sigma_{\mu\nu}D^\mu F^{\nu\alpha}\Big] - \frac{1}{4\hat{m}_{Q'}^2}\Big[(-iv'\cdot\overleftarrow{D} - \bar{\Lambda})\Big(\overleftarrow{D}_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big) + iF^{\alpha\beta}v'_\alpha\overleftarrow{D}_\beta\\
&-F^{\nu\alpha}\overleftarrow{D}^\mu v'_\alpha\sigma_{\mu\nu}\Big]\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma + \frac{1}{4\hat{m}_Q^2}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\Big(D_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big)\frac{P_+}{\bar{\Lambda}+iv\cdot D}\\
&\Big(D_\perp^2 + \frac{i}{2}\sigma_{\gamma\delta}F^{\gamma\delta}\Big) + \frac{1}{4\hat{m}_{Q'}^2}\Big(\overleftarrow{D}_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big)\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Big(\overleftarrow{D}_\perp^2 + \frac{i}{2}\sigma_{\gamma\delta}F^{\gamma\delta}\Big)\\
&\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma + \frac{1}{4\hat{m}_Q\hat{m}_{Q'}}\Big(\overleftarrow{D}_\perp^2 + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\Big)\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\\
&\Big(D_\perp^2 + \frac{i}{2}\sigma_{\gamma\delta}F^{\gamma\delta}\Big) + O\Big(1/\hat{m}_{Q^{(')}}^3\Big)\Big\}Q_v^+|H_v\rangle
\end{aligned} \tag{55}$$

with $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^\alpha, \gamma^\beta]$, $P'_+ = \frac{1+\not{v}'}{2}$, and the gluon field strength tensor $F^{\alpha\beta} = [D^\beta, D^\alpha]$. Then a set of heavy flavor and spin independent wave functions can be introduced as follows,

$$
\begin{aligned}
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma Q^+_v|M_v\rangle = -\xi(\omega)Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{-P_+}{\bar{\Lambda}+iv\cdot D}D^2_\perp Q^+_v|M_v\rangle = -\kappa_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{-1}{\bar{\Lambda}+iv\cdot D}P_+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}Q^+_v|M_v\rangle = \frac{1}{\bar{\Lambda}}Tr[\kappa_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}[D^2_\perp(iv\cdot D-\bar{\Lambda}) - iv_\mu D_\nu F^{\mu\nu}]Q^+_v|M_v\rangle = -\varrho_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\Big[\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}(iv\cdot D-\bar{\Lambda}) + v_\alpha\sigma_{\mu\nu}D^\mu F^{\nu\alpha}\Big]Q^+_v|M_v\rangle\\
&\qquad\qquad = \frac{1}{\bar{\Lambda}}Tr[\varrho_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}D^2_\perp\frac{P_+}{\bar{\Lambda}+iv\cdot D}D^2_\perp Q^+_v|M_v\rangle = -\chi_1(\omega)\frac{1}{\bar{\Lambda}^2}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\Big[D^2_\perp\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta} + \frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P_+}{\bar{\Lambda}+iv\cdot D}D^2_\perp\Big]Q^+_v|M_v\rangle\\
&\qquad\qquad = \frac{1}{\bar{\Lambda}^2}Tr[\chi_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\gamma\delta}F^{\gamma\delta}Q^+_v|M_v\rangle\\
&\qquad\qquad = -\frac{1}{\bar{\Lambda}^2}Tr[\chi_{\alpha\beta\gamma\delta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}P_+\frac{i}{2}\sigma^{\gamma\delta}\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\overleftarrow{D}^2_\perp\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}D^2_\perp Q^+_v|M_v\rangle = -\eta_1(\omega)\frac{1}{\bar{\Lambda}^2}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\overleftarrow{D}^2_\perp\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}Q^+_v|M_v\rangle\\
&\qquad\qquad = \frac{1}{\bar{\Lambda}^2}Tr[\eta_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}],\\
&\langle M'_{v'}|\bar{Q}'^{+}_{v'}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\gamma\delta}F^{\gamma\delta}Q^+_v|M_v\rangle\\
&\qquad\qquad = -\frac{1}{\bar{\Lambda}^2}Tr[\eta_{\alpha\beta\gamma\delta}(v,v')\bar{\mathcal{M}}'\frac{i}{2}\sigma^{\alpha\beta}P'_+\Gamma P_+\frac{i}{2}\sigma^{\gamma\delta}\mathcal{M}], \qquad (56)
\end{aligned}
$$

where $\mathcal{M}$ is the spin wave function

$$
\mathcal{M}(v) = \sqrt{\bar{\Lambda}}P_+\left\{\begin{array}{ll} -\gamma^5 & \text{for pseudoscalar meson} \\ \not{\epsilon} & \text{for vector meson}\end{array}\right. \qquad (57)
$$

and $\bar{\mathcal{M}} \equiv \gamma^0\mathcal{M}^\dagger\gamma^0$. The decomposition of the tensors $\kappa_{\alpha\beta}(v,v')$, $\varrho_{\alpha\beta}(v,v')$, $\chi_{\alpha\beta}(v,v')$, $\eta_{\alpha\beta}(v,v')$, $\chi_{\alpha\beta\gamma\delta}(v,v')$, and $\eta_{\alpha\beta\gamma\delta}(v,v')$ are the same as that presented in the Appendix B of Ref.[17]. For simplicity, when the variable ω is not written explicitly, we refer to the zero recoil values of relevant functions, i.e. $h_{A_1} = h_{A_1}(1)$, $\kappa_1 = \kappa_1(1)$, etc.

With the definition in Eqs.(54) and (56), one obtains from Eqs.(37) and (55)

$$
\begin{aligned}
\bar{\Lambda}_{D(B)} &= \bar{\Lambda} - \frac{1}{\hat{m}_{c(b)}}(\kappa_1 + 3\kappa_2) - \frac{1}{2\hat{m}_{c(b)}^2\bar{\Lambda}}(\varrho_1\bar{\Lambda} - 3\varrho_2\bar{\Lambda} - \chi_1 - 3\chi_2 + 3\chi_4 \\
&\quad +9\chi_5 + 6\chi_6) + \frac{1}{4\hat{m}_{c(b)}^2\bar{\Lambda}}(\eta_1 + 6\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6) + O\Big(\frac{1}{\hat{m}_{c(b)}^3}\Big), \qquad (58) \\
\bar{\Lambda}_{D^*(B^*)} &= \bar{\Lambda} - \frac{1}{\hat{m}_{c(b)}}(\kappa_1 - \kappa_2) - \frac{1}{2\hat{m}_{c(b)}^2\bar{\Lambda}}(\varrho_1\bar{\Lambda} - \varrho_2\bar{\Lambda} - \chi_1 + \chi_2 \\
&\quad +3\chi_4 + \chi_5 - 2\chi_6) + \frac{1}{4\hat{m}_{c(b)}^2\bar{\Lambda}}(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + O\Big(\frac{1}{\hat{m}_{c(b)}^3}\Big), \qquad (59)
\end{aligned}
$$

where the normalization of the Isgur-Wise function $\xi(1) = 1$ [3] has been used.

At the zero recoil point, Eqs.(37) and (54)-(59) yield

$$
\begin{aligned}
h_+ &= 1 + \frac{1}{8\bar{\Lambda}^2}\Big(\frac{1}{\hat{m}_b} - \frac{1}{\hat{m}_c}\Big)^2\Big[(\kappa_1 + 3\kappa_2)^2 - \eta_1 - 6\eta_2 + 3\eta_4 + 9\eta_5 + 6\eta_6\Big], \\
h_{A_1} &= 1 + \frac{1}{8\bar{\Lambda}^2}\Big[\frac{1}{\hat{m}_b}(\kappa_1 + 3\kappa_2) - \frac{1}{\hat{m}_c}(\kappa_1 - \kappa_2)\Big]^2 - \frac{1}{8\hat{m}_b^2\bar{\Lambda}^2}(\eta_1 + 6\eta_2 - 3\eta_4 - 9\eta_5 - 6\eta_6) \\
&\quad - \frac{1}{8\hat{m}_c^2\bar{\Lambda}^2}(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) + \frac{1}{4\hat{m}_b\hat{m}_c\bar{\Lambda}^2}(\eta_1 + 2\eta_2 + \eta_4 + 3\eta_5 + 2\eta_6), \\
h_1 &= 1 + \frac{1}{8\bar{\Lambda}^2}\Big(\frac{1}{\hat{m}_b} - \frac{1}{\hat{m}_c}\Big)^2\Big[(\kappa_1 - \kappa_2)^2 - \eta_1 + 2\eta_2 + 3\eta_4 + \eta_5 - 2\eta_6\Big], \\
h_7 &= -\Big[1 + \frac{1}{8\bar{\Lambda}^2}\Big(\frac{1}{\hat{m}_b} - \frac{1}{\hat{m}_c}\Big)^2(\kappa_1 - \kappa_2)^2 - \frac{1}{8\bar{\Lambda}^2}\Big(\frac{1}{\hat{m}_b^2} + \frac{1}{\hat{m}_c^2}\Big)(\eta_1 - 2\eta_2 - 3\eta_4 - \eta_5 + 2\eta_6) \\
&\quad + \frac{1}{4\hat{m}_b\hat{m}_c\bar{\Lambda}^2}(\eta_1 - 2\eta_2 + \eta_4 - \eta_5 - 2\eta_6)\Big]. \qquad (60)
\end{aligned}
$$

Then it is clear that all matrix elements in (54) are protected from $1/\hat{m}_Q$ order corrections at zero recoil. Furthermore, one has $h_-(\omega) = h_2(\omega) = 0$ [17] because in the new framework of HQEFT the effective current $J_{Q,v}^{eff}$ contains only terms with even powers of $\not{D}_\perp$.

IV. $|V_{cb}|$ FROM EXCLUSIVE B DECAYS

The $B \to D^*(D)l\nu$ differential decay rates are

$$
\begin{aligned}
\frac{d\Gamma(B \to D^* l\nu)}{d\omega} &= \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3\sqrt{\omega^2 - 1}(\omega + 1)^2 \\
&\quad \times\Big[1 + \frac{4\omega}{\omega + 1}\frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\Big]|V_{cb}|^2\mathcal{F}^2(\omega), \qquad (61) \\
\frac{d\Gamma(B \to D l\nu)}{d\omega} &= \frac{G_F^2}{48\pi^3}(m_B + m_D)^2 m_D^3(\omega^2 - 1)^{3/2}|V_{cb}|^2\mathcal{G}^2(\omega) \qquad (62)
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{F}(1) &= \eta_A h_{A_1}(1), \qquad (63) \\
\mathcal{G}(1) &= \eta_V\Big[h_+(1) - \frac{m_B - m_D}{m_B + m_D}h_-(1)\Big], \qquad (64)
\end{aligned}
$$

where the QCD radiative corrections to two loops give the short distance coefficients $\eta_A = 0.960 \pm 0.007$ and $\eta_V = 1.022 \pm 0.004$ [30].

The form factors h_i contain long distance effects and can be estimated by nonperturbative methods such as lattice simulations, QCD sum rules or quark models. Here we do not perform such calculations but try to make model independent prediction on $|V_{cb}|$ using the HQE discussed in the previous sections. Suppose that the residual momenta of the heavy quarks approximately equal and the longitudinal residual momenta of dressed heavy quarks be much smaller than the binding energy, we then have in a good approximation

$$\frac{1}{i\not{D}_\parallel + \bar{\Lambda}} \sim \frac{1}{-i \overleftarrow{\not{D}}_\parallel + \bar{\Lambda}} \sim \frac{1}{\bar{\Lambda}} \tag{65}$$

which implies $O_3(\Gamma) \sim O_3'(\Gamma) \sim O_4(\Gamma)$. Consequently, we arrive at the following relations among the wave functions:

$$\chi_1 = \eta_1, \quad \chi_2 = 2\eta_2, \quad \chi_i = \eta_i \ (i = 4, 5, 6), \tag{66}$$

which will be adopted in the following discussions.

Since the contribution of the chromomagnetic moment operator is generally much smaller than that of the kinetic energy operator, we neglect operators containing two field strength tensors of gluon but remain those containing only one. As a result, χ_j and η_j for $j = 4, 5, 6$ will be neglected. Thus we get from Eqs.(58)-(60)

$$\bar{\Lambda}_{D(B)} = \bar{\Lambda} - \frac{1}{\hat{m}_{c(b)}}(\kappa_1 + 3\kappa_2) - \frac{1}{4\hat{m}^2_{c(b)}\bar{\Lambda}}(F_1 + 3F_2) + O\Big(\frac{1}{\hat{m}^3_{c(b)}}\Big), \tag{67}$$

$$\bar{\Lambda}_{D^*(B^*)} = \bar{\Lambda} - \frac{1}{\hat{m}_{c(b)}}(\kappa_1 - \kappa_2) - \frac{1}{4\hat{m}^2_{c(b)}\bar{\Lambda}}(F_1 - F_2) + O\Big(\frac{1}{\hat{m}^3_{c(b)}}\Big) \tag{68}$$

and

$$\begin{aligned} h_{A_1} &= 1 + \frac{1}{8\bar{\Lambda}^2}\Big[\frac{\kappa_1 + 3\kappa_2}{\hat{m}_b} - \frac{\kappa_1 - \kappa_2}{\hat{m}_c}\Big]^2 - \frac{1}{24\hat{m}_b^2\bar{\Lambda}^2}(2\bar{\Lambda}\varrho_1 + 6\bar{\Lambda}\varrho_2 - F_1 - 3F_2) \\ &\quad - \frac{1}{24\hat{m}_c^2\bar{\Lambda}^2}(2\bar{\Lambda}\varrho_1 - 2\bar{\Lambda}\varrho_2 - F_1 + F_2) + \frac{1}{12\hat{m}_b\hat{m}_c\bar{\Lambda}^2}(2\bar{\Lambda}\varrho_1 + 2\bar{\Lambda}\varrho_2 - F_1 - F_2), \end{aligned} \tag{69}$$

$$h_+ = 1 + \frac{1}{8\bar{\Lambda}^2}\Big(\frac{1}{\hat{m}_b} - \frac{1}{\hat{m}_c}\Big)^2\Big[(\kappa_1 + 3\kappa_2)^2 - \frac{1}{3}(2\bar{\Lambda}\varrho_1 + 6\bar{\Lambda}\varrho_2 - F_1 - 3F_2)\Big], \tag{70}$$

$$h_- = 0, \tag{71}$$

where F_1 and F_2 are defined as

$$F_1 = 2\bar{\Lambda}\varrho_1 - 3\eta_1, \qquad F_2 = 2\bar{\Lambda}\varrho_2 - 6\eta_2. \tag{72}$$

As already mentioned in the previous section, the form factors h_{A_1} and h_+ are protected from $1/\hat{m}_Q$ order correction, and $h_- = 0$ holds up to order $1/\hat{m}_Q^2$ in our expansion. These make both the semileptonic decays of $B \to D^*\ell\nu$ and $B \to D\ell\nu$ the appropriate channels for the $|V_{cb}|$ extraction. From Eqs.(67) and (68) one can estimate the zero recoil values of κ_i and F_i from the bottom and charm meson masses ($m_B = 5.279$GeV, $m_{B^*} = 5.325$GeV, $m_D = 1.865$GeV and $m_{D^*} = 2.007$GeV). κ_i and F_i as functions of the variables $\hat{m}_b$ and

$\hat{m}_b - \hat{m}_c$ are shown in Fig.1. It is found that κ_1 and κ_2 are independent of $\bar{\Lambda}$, and the change of $\bar{\Lambda}$ value only affects F_1 and F_2 quite slightly. κ_1 is sensitive to $\hat{m}_b$ and also influenced by $\hat{m}_b - \hat{m}_c$. κ_2 changes slightly against $\hat{m}_b - \hat{m}_c$ but is almost independent of $\hat{m}_b$. Both F_1 and F_2 heavily depend on $\hat{m}_b - \hat{m}_c$, and F_1 is also sensitive to $\hat{m}_b$. When taking

$$\hat{m}_b = 5.23 \sim 5.27\text{GeV}, \quad \hat{m}_b - \hat{m}_c = 3.45 \sim 3.55\text{GeV}, \quad \bar{\Lambda} = 0.50 \sim 0.56\text{GeV}, \tag{73}$$

we obtain

$$\begin{aligned} &\kappa_1 \approx -0.31\text{GeV}^2, \quad \kappa_2 \approx 0.06\text{GeV}^2, \\ &F_1 \approx -0.30\text{GeV}^4, \quad\ \ F_2 \approx 0.01\text{GeV}^4. \end{aligned} \tag{74}$$

The data in Eqs.(73) and (74) are consistent with the results in Ref.[24]. In that reference $\kappa_1 = -0.50 \pm 0.18\text{GeV}^2$, $\bar{\Lambda} = 0.53 \pm 0.08\text{GeV}$ and $\kappa_1 \approx -0.43\text{GeV}^2$, $\kappa_2 \approx 0.08\text{GeV}^2$ are given via different methods of analysis on the sum rule equations that include only one-loop perturbative contributions. When the two-loop perturbative contributions are considered in the sum rule for the decay constant, $\kappa_1 \approx -0.34\text{GeV}^2$, $\bar{\Lambda} = 0.56 \pm 0.08\text{GeV}$ are obtained.

h_{A_1} and h_+ in (69) and (70) also depend on ϱ_1 and ϱ_2. Note that the definition of κ_i and ϱ_i in (56) can be written as

$$\begin{aligned} \langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{1}{\bar{\Lambda}+iv\cdot D}(i\not{D}_\perp)^2 Q_v^+|M_v\rangle &= -\kappa_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}] \\ &+\frac{1}{\bar{\Lambda}}Tr[\kappa_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}], \end{aligned} \tag{75}$$

$$\begin{aligned} \langle M'_{v'}|\bar{Q}'^{+}_{v'}\Gamma\frac{1}{\bar{\Lambda}+iv\cdot D}(i\not{D}_\perp)(i\not{D}_\parallel+\bar{\Lambda})(i\not{D}_\perp) Q_v^+|M_v\rangle &= -\varrho_1(\omega)\frac{1}{\bar{\Lambda}}Tr[\bar{\mathcal{M}}'\Gamma\mathcal{M}] \\ &+\frac{1}{\bar{\Lambda}}Tr[\varrho_{\alpha\beta}(v,v')\bar{\mathcal{M}}'\Gamma P_+\frac{i}{2}\sigma^{\alpha\beta}\mathcal{M}]. \end{aligned} \tag{76}$$

Then the approximation (65) implies $\frac{\varrho_i}{\bar{\Lambda}\kappa_i} \approx 1$. The resulting $|V_{cb}|$ value is shown in Figs.2-5. Using (73) and allowing ϱ_i change in the range

$$\frac{\varrho_i}{\bar{\Lambda}\kappa_i} = 0 \sim 2 \quad (i=1,2), \tag{77}$$

we get

$$h_{A_1}(1) \ = \ 1.014 \pm 0.034, \tag{78}$$
$$h_+(1) \ = \ 0.997 \pm 0.025. \tag{79}$$

Consequently, the averages of measurements [31]

$$|V_{cb}|\mathcal{F}(1) \ = \ 0.0360 \pm 0.0013, \tag{80}$$
$$|V_{cb}|\mathcal{G}(1) \ = \ 0.039 \pm 0.004 \tag{81}$$

give

$$|V_{cb}|_{B\to D^*} \ = \ 0.0370 \pm 0.0013_{\text{exp}} \pm 0.0015_{\text{th}}, \tag{82}$$
$$|V_{cb}|_{B\to D} \ = \ 0.0383 \pm 0.0039_{\text{exp}} \pm 0.0011_{\text{th}}. \tag{83}$$

So the $|V_{cb}|$ values extracted from $B \to D^*\ell\nu$ and $B \to D\ell\nu$ decays are consistent within the errors of experimental data. It is noticed that the value extracted from $B \to D\ell\nu$ suffers from relatively larger experimental uncertainty, which can be seen in Eq.(81). The result for $|V_{cb}|$ in (82) is marginally consistent with the value given in Ref.[31] (0.0386 ± 0.0013) but has a smaller center value. For more precise determination of $|V_{cb}|$, it would be helpful to evaluate $1/\hat{m}_Q^2$ order wave functions such as F_i and ϱ_i through other methods like lattice or QCD sum rule calculation.

V. CONCLUSIONS

We have briefly reviewed the derivation of a heavy quark effective field theory. This HQEFT is complete in that the effective Lagrangian contains the heavy quark-antiquark coupling terms, which appear as finite mass corrections. Unlike the usual naive heavy quark-hadron duality, we do not simply treat the light components of hadrons as spectators. Instead, the flavor independent nonperturbative effects of light degrees of freedom are attributed to the dressed heavy quark with the dressed mass $\hat{m}_Q = m_Q + \bar{\Lambda}$ and the total momentum $p_Q = \hat{m}_Q v + \tilde{k}$, and it is such a dressed heavy quark that dualizes the heavy hadron. Consequently, HQEFT has been extended into the formulation in terms of $1/\hat{m}_Q$ expansion. Such an expansion is consistent with the picture of dressed heavy quark and becomes more convergent. The HQE of heavy-to-heavy transition matrix elements has been consistently extended into a $1/\hat{m}_Q$ expansion form, in which the contribution of heavy antiquark (or quark) field is integrated into the effective current.

$|V_{cb}|$ extraction from $B \to D^*(D)\ell\nu$ decays has been discussed by using the HQE in $1/\hat{m}_Q$. Due to the appropriate definition of effective states, zero recoil values of the relevant form factors can be estimated from the hadron masses. Using some approximate relations between wave functions $|V_{cb}|$ is found to be $0.0370 \pm 0.0013_{\rm exp} \pm 0.0015_{\rm th}$ from $B \to D^*\ell\nu$ decay and $0.0383 \pm 0.0039_{\rm exp} \pm 0.0011_{\rm th}$ from $B \to D\ell\nu$ decay. For these two channels, experimental study of $B \to D\ell\nu$ is more difficult and it has larger uncertainties. Nevertheless, the current data of $|V_{cb}|\mathcal{G}(1)$ and the form factors extracted within the framework of HQEFT give the $|V_{cb}|$ value consistent with that from $B \to D^*\ell\nu$ decay, which shows the reliability of the $1/\hat{m}_Q$ expansion in this application.

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Appendix A

This appendix is devoted to the derivation of the effective current $J^{eff}_{Q,v}$ in Eqs.(48) and (51). In particular, we would like to show why one can use (50) as the propagator of Q_v field.

Firstly, one may notice that the gluon couplings arising from $\not{D}_{\parallel}$ can be trivialized by the

Wilson-line transformation [32]. One can introduce new field variable Q_v^0 by [19]

$$Q_v = \mathcal{P}e^{ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} Q_v^0 \equiv W(x,v)Q_v^0, \tag{A1}$$

$$\bar{Q}_v = \bar{Q}_v^0 \mathcal{P}e^{-ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} \equiv \bar{Q}_v^0 W^{-1}(x,v), \tag{A2}$$

where $\mathcal{P}$ denotes path ordering with $x^\mu = v^\mu \tau$. Since

$$v\cdot DQ_v = \mathcal{P}e^{ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} v\cdot \partial Q_v^0, \tag{A3}$$

$$(\not{D} - \not{v} v\cdot D)Q_v = \mathcal{P}e^{ig\int_{-\infty}^{v\cdot x} d\tau v\cdot A^a T^a} (\not{D} - \not{v} v\cdot D)Q_v^0, \tag{A4}$$

one can write Eqs.(44)-(47) as

$$J_{Q,v}^{(0)}(x) = \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v' - \sigma\hat{m}_Q v)\cdot x} \bar{Q}_{v'}^{0\prime}(x) \frac{1+\sigma' \not{v}'}{2} W^{-1}(x,v')\Gamma W(x,v) \frac{1+\sigma \not{v}}{2} Q_v^0(x), \tag{A5}$$

$$\begin{aligned} J_{Q,v}^{(1/\hat{m}_Q)}(x) = & \sum_{\sigma,\sigma'=\pm 1} e^{i(\sigma'\hat{m}_{Q'}v' - \sigma\hat{m}_Q v)\cdot x} \bar{Q}_{v'}^{0\prime}(x) \frac{1+\sigma' \not{v}'}{2} \Big[\frac{1}{2\hat{m}_Q} W^{-1}(x,v')\Gamma W(x,v) i\not{D}_\perp \\ & + \frac{1}{2\hat{m}_{Q'}} (-i\overleftarrow{\not{D}_\perp}) W^{-1}(x,v')\Gamma W(x,v) + \frac{1}{4\hat{m}_Q^2} W^{-1}(x,v')\Gamma W(x,v) \\ & (i\not{v} v\cdot\partial + \bar{\Lambda}) i\not{D}_\perp + \frac{1}{4\hat{m}_{Q'}^2} (-i\overleftarrow{\not{D}_\perp})(-i\not{v} v\cdot \overleftarrow{\partial} + \bar{\Lambda}') W^{-1}(x,v')\Gamma W(x,v) \\ & + \frac{1}{4\hat{m}_Q \hat{m}_{Q'}} (-i\overleftarrow{\not{D}_\perp}) W^{-1}(x,v')\Gamma W(x,v)(i\not{D}_\perp) \\ & + O\Big(\frac{1}{\hat{m}_{Q^{(\prime)}}^3}\Big)\Big] \frac{1+\sigma \not{v}}{2} Q_v^0(x) \end{aligned} \tag{A6}$$

and

$$\begin{aligned} \mathcal{L}_{Q,v}^{I(1/\hat{m}_Q)} = & \sum_{\varepsilon=\pm 1} \bar{Q}_v^0 \frac{1+\varepsilon \not{v}}{2} \Big[\frac{(i\not{D}_\perp)^2}{2\hat{m}_Q} + \frac{1}{4\hat{m}_Q^2} i\not{D}_\perp (i\not{v} v\cdot\partial + \bar{\Lambda}) i\not{D}_\perp \\ & + O\Big(\frac{1}{\hat{m}_Q^3}\Big)\Big] \frac{1+\varepsilon \not{v}}{2} Q_v^0, \end{aligned} \tag{A7}$$

$$\begin{aligned} \mathcal{L}_{Q,v}^{II(1/\hat{m}_Q)} = & \sum_{\varepsilon=\pm 1} e^{2i\hat{m}_Q v\cdot x\varepsilon} \bar{Q}_v^0 \frac{1+\varepsilon \not{v}}{2} \Big[\frac{1}{2\hat{m}_Q} (-i\not{v} v\cdot \overleftarrow{\partial} + \bar{\Lambda}) i\not{D}_\perp \\ & + \frac{1}{4\hat{m}_Q^2} (-i\not{v} v\cdot \overleftarrow{\partial} + \bar{\Lambda})(i\not{v} v\cdot\partial + \bar{\Lambda}) i\not{D}_\perp \\ & + \frac{1}{4\hat{m}_Q^2} (-i\overleftarrow{\not{D}_\perp})^2 i\not{D}_\perp + O\Big(\frac{1}{\hat{m}_Q^3}\Big)\Big] \frac{1-\varepsilon \not{v}}{2} Q_v^0. \end{aligned} \tag{A8}$$

In terms of Q_v^0, the effective Lagrangian in heavy quark limit turns into

$$\mathcal{L}_{Q,v}^{(0)} = \bar{Q}_v^0 (i\not{v} v\cdot\partial + \bar{\Lambda}) Q_v^0, \tag{A9}$$

and the contraction of Q_v^0 fields yields the propagator:

$$
\begin{aligned}
\underbrace{Q_v^0(x), \quad \bar{Q}_v^0(y)} &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i}{\not{v}\, v\cdot k + \bar{\Lambda}} \\
&= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i}{\bar{\Lambda}} \sum_{n=0}^{\infty} \Big(-\not{v} \frac{v\cdot k}{\bar{\Lambda}} \Big)^n. \qquad \text{(A10)}
\end{aligned}
$$

To illustrate the details of deriving the effective current $J^{eff}_{Q,v}$, we consider as an example the two-point correlation function

$$
\begin{aligned}
&\int d^4y T\{\bar{Q}_{v'}(x)\hat{O}_1(\Gamma_1)_{(x)} Q_v(x), \bar{Q}_v(y)\hat{O}_2(\Gamma_2)_{(y)} Q_{v''}(y)\} \qquad \text{(A11)} \\
&= \int d^4y T\{\bar{Q}^0_{v'}(x)\hat{O}^0_1(W^{-1}(x,v')\Gamma_1 W(x,v))_{(x)} Q^0_v(x), \\
&\qquad \bar{Q}^0_v(y)\hat{O}^0_2(W^{-1}(y,v)\Gamma_2 W(y,v''))_{(y)} Q^0_{v''}(y)\}, \qquad \text{(A12)}
\end{aligned}
$$

where $\hat{O}_1(\Gamma)$ and $\hat{O}_2(\Gamma)$ can be any local operators that may contain $\overleftarrow{\not{D}}_\parallel$ ($\not{D}_\parallel$) and $\overleftarrow{\not{D}}_\perp$ ($\not{D}_\perp$) on the left (right) of the Dirac matrixes Γ. $\hat{O}^0_1(\Gamma)$ and $\hat{O}^0_2(\Gamma)$ are obtained from $\hat{O}_1(\Gamma)$ and $\hat{O}_2(\Gamma)$ by replacing $\overleftarrow{\not{D}}_\parallel$ and $\not{D}_\parallel$ in the operators with $\not{v} v\cdot\overleftarrow{\partial}$ and $\not{v} v\cdot\partial$, respectively.

Using the propagator (A10) for field contraction and applying the integration by parts, the two-point function (A11) becomes

$$
\begin{aligned}
&\int d^4y \bar{Q}^0_{v'}(x)\hat{O}^0_1(W^{-1}(x,v')\Gamma_1 W(x,v))_{(x)} \int \frac{d^4k}{(2\pi)^4} \frac{i}{\bar{\Lambda}} \sum_{n=0}^{\infty} \Big[\Big(\frac{i}{\bar{\Lambda}} \not{v} v\cdot\partial_{(y)} \Big)^n e^{-ik\cdot(x-y)} \Big] \\
&\hat{O}^0_2(W^{-1}(y,v)\Gamma_2 W(y,v''))_{(y)} Q^0_{v''}(y) \\
= &\int d^4y \bar{Q}^0_{v'}(x)\hat{O}^0_1(W^{-1}(x,v')\Gamma_1 W(x,v))_{(x)} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i}{\bar{\Lambda}} \sum_{n=0}^{\infty} \Big(-\frac{i}{\bar{\Lambda}} \not{v} v\cdot\partial_{(y)} \Big)^n \\
&\hat{O}^0_2(W^{-1}(y,v)\Gamma_2 W(y,v''))_{(y)} Q^0_{v''}(y) \\
= &\ \bar{Q}^0_{v'}(x)\hat{O}^0_1(W^{-1}(x,v')\Gamma_1 W(x,v))_{(x)} \frac{i}{\bar{\Lambda}} \sum_{n=0}^{\infty} \Big(-\frac{i}{\bar{\Lambda}} \not{v} v\cdot\partial_{(x)} \Big)^n \\
&\hat{O}^0_2(W^{-1}(x,v)\Gamma_2 W(x,v''))_{(x)} Q^0_{v''}(x) \\
= &\ \bar{Q}_{v'}(x)\hat{O}_1(\Gamma_1)_{(x)} \frac{i}{\bar{\Lambda}} \sum_{n=0}^{\infty} \Big(-\frac{i}{\bar{\Lambda}} \not{D}_{\parallel(x)} \Big)^n \hat{O}_2(\Gamma_2)_{(x)} Q_{v''}(x) \\
= &\ \bar{Q}_{v'}(x)\hat{O}_1(\Gamma_1)_{(x)} \frac{i}{i\not{D}_{\parallel(x)} + \bar{\Lambda}} \hat{O}_2(\Gamma_2)_{(x)} Q_{v''}(x), \qquad \text{(A13)}
\end{aligned}
$$

the final expression of which can be obtained directly from (A11) with using (50) as Q_v field propagator.

With the same techniques it is then easy to derive Eq.(48) from (44)-(47).

[1] E. V. Shuryak, Phys. Lett. **B 93**, 134 (1980); Nucl. Phys. **B 198**, 83 (1982).

[2] S. Nussinov and W. Wetzel, Phys. Rev. **D 36**, 130 (1987).
[3] N. Isgur and M. Wise, Phys. Lett. **B 232**, 113 (1989); **B 237**, 527 (1990); **B 206**, 681 (1988).
[4] M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); **47**, 199 (1988).
[5] H. Georgi, Phys. Lett. **B 240**, 447 (1990).
[6] T. Mannel and Z. Ryzak, Phys. Lett. **B 247**, 2388 (1990).
[7] M. E. Luke, Phys. Lett. **B 252**, 447 (1990).
[8] B. Grinstein, Nucl. Phys. **B 339**, 253 (1990).
[9] A. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. **B 343**, 1 (1990).
[10] A. F. Falk, B. Grinstein and M. E. Luke, Nucl. Phys. **B 357**, 185 (1991).
[11] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B 368**, 204 (1992).
[12] B. Grinstein, SSCL-Preprint-34, 1992.
[13] T. Mannel, Phys. Rev. **D 50**, 428 (1994).
[14] T. Mannel, Nucl. Phys. **B 413**, 396 (1994).
[15] M. Neubert, Phys. Rept. **245**, 259 (1994).
[16] Y. L. Wu, Mod. Phys. Lett. **A 8**, 819 (1993).
[17] W. Y. Wang, Y. L. Wu and Y. A. Yan, Int. J. Mod. Phys. **A 15**, 1817 (2000).
[18] Y. L. Wu, Y. A. Yan, M. Zhong, Y. B. Zuo and W. Y. Wang, Mod. Phys. Lett. **A 18**, 1303 (2003).
[19] Y. L. Wu, Int. J. Mod. Phys. **A 21**, 5743 (2006); and references therein.
[20] Y. A. Yan, Y. L. Wu and W. Y. Wang, Int. J. Mod. Phys. **A 15**, 2735 (2000).
[21] Y. L. Wu and Y. A. Yan, Int. J. Mod. Phys. **A 16**, 285 (2001).
[22] Y. B. Zuo, Y. A. Yan, Y. L Wu and W. Y. Wang, Int. J. Mod. Phys. **A 19**, 3685 (2004).
[23] W. Y. Wang, Y. L. Wu, Y. A. Yan, M. Zhong and Y. B. Zuo, Mod. Phys. Lett. **A 19**, 1379 (2004).
[24] W. Y. Wang and Y. L. Wu, Int. J. Mod. Phys. **A 16**, 377 (2001).
[25] W. Y. Wang and Y. L. Wu, Phys. Lett. **B 515**, 57 (2001).
[26] W. Y. Wang and Y. L. Wu, Phys. Lett. **B 519**, 219 (2001).
[27] W. Y. Wang and Y. L. Wu, M. Zhong, J. Phys. **G 29**, 2743 (2003).
[28] W. Y. Wang, Y. L. Wu and M. Zhong, Phys. Rev. **D 67**, 014024 (2003).
[29] W. Y. Wang, Y. L. Wu and M. Zhong, Phys. Lett. **B 628**, 228 (2005).
[30] A. Czarnecki, Phys. Rev. Lett. **76**, 4124 (1996).
[31] Particle Data Group, C. Amsler *et al.*, Phys. Lett. **B 667**, 1 (2008).
[32] F. Hussain, J. G. Körner, K. Schilcher, G. Thompson and Y. L. Wu, Phys. Lett. **B 249**, 295 (1990).

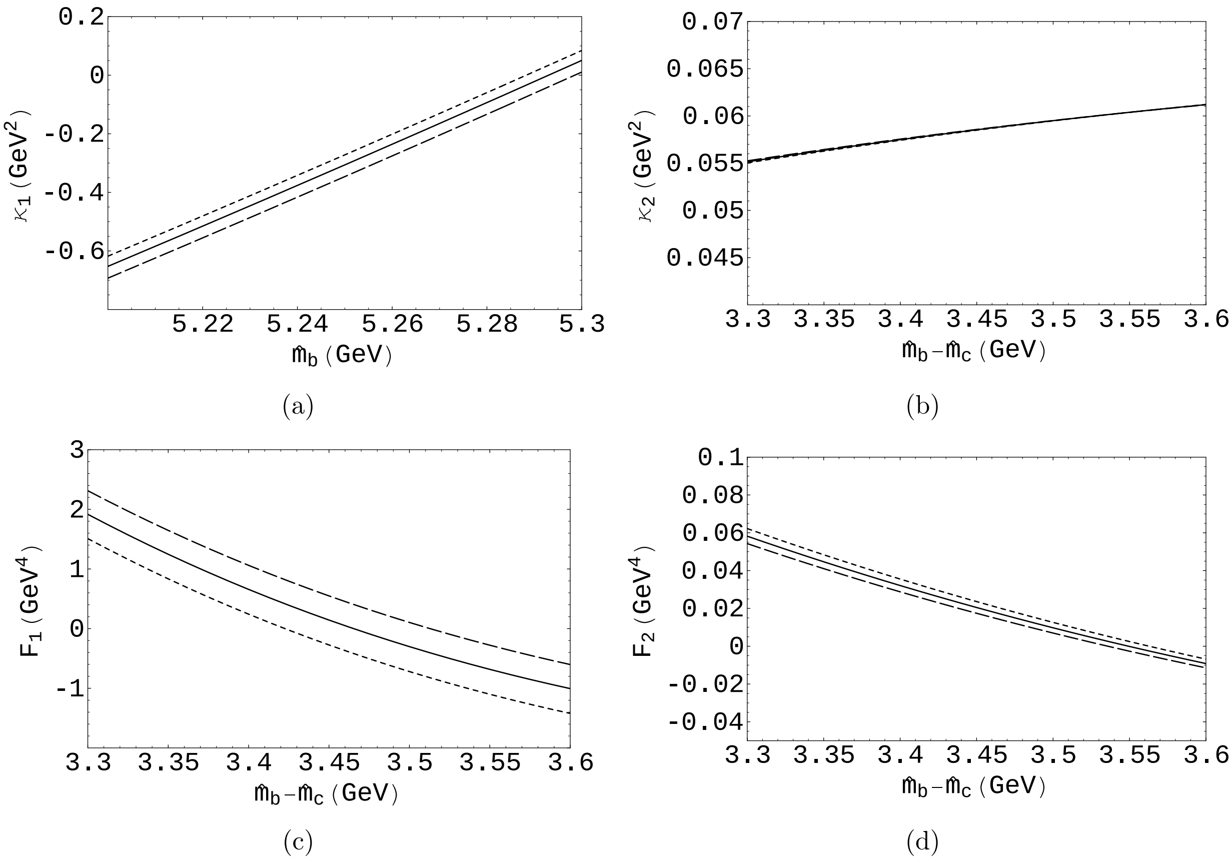


FIG. 1: κ_i and F_i as functions of the variables $\hat{m}_b$ and $\hat{m}_b - \hat{m}_c$. The dashed, solid and dotted curves correspond to $\hat{m}_b - \hat{m}_c$ =3.45, 3.50 and 3.55GeV in (a); and $\hat{m}_b$ =5.23, 5.25 and 5.27GeV in (b)-(d). Figures (c) and (d) are obtained at $\bar{\Lambda} = 0.53$GeV.

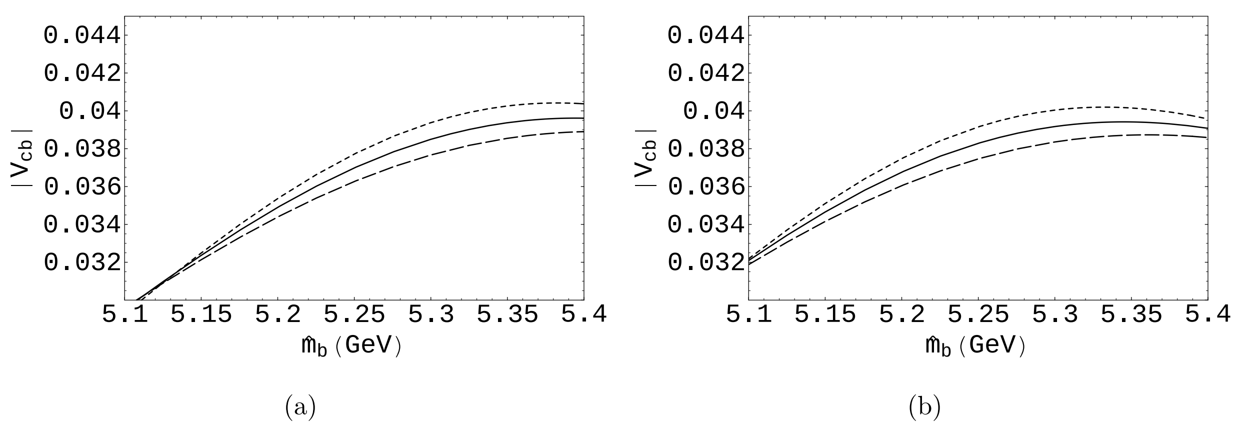
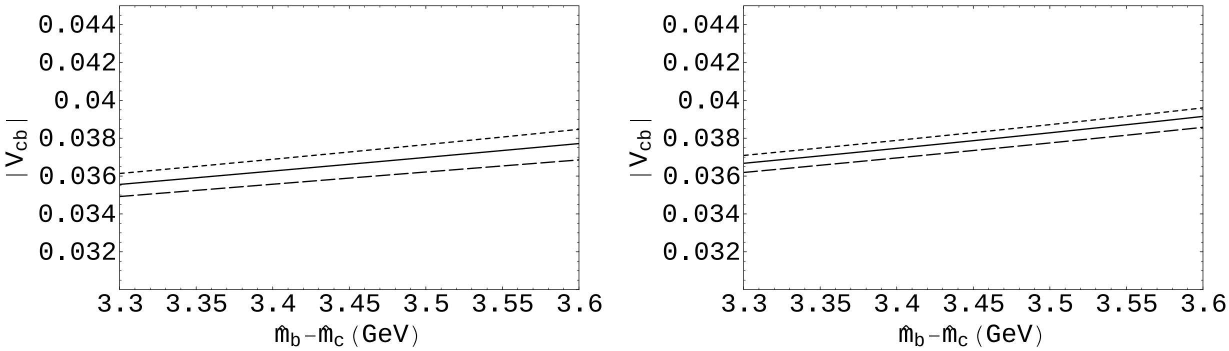
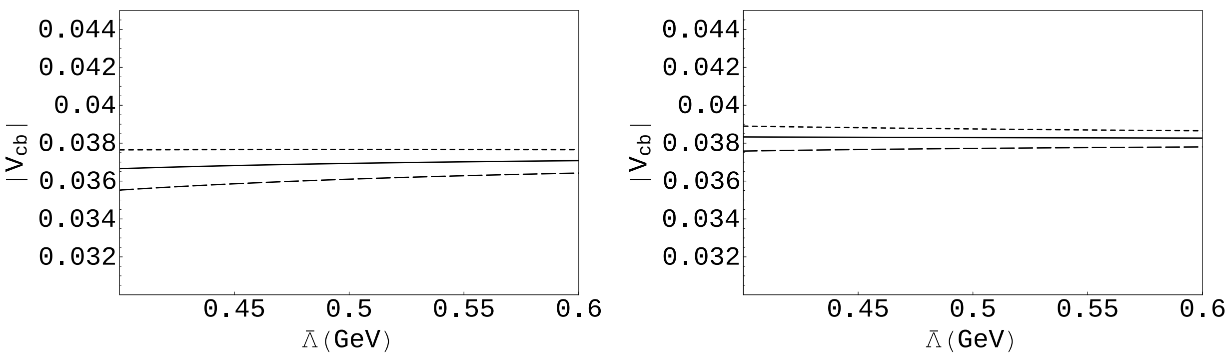


FIG. 2: $|V_{cb}|$ extracted from $B \to D^* \ell \nu$ (a) and $B \to D \ell \nu$ (b). The dashed, solid and dotted curves correspond to $\hat{m}_b - \hat{m}_c$ =3.4, 3.5 and 3.6GeV, respectively. $\bar{\Lambda} = 0.53$GeV and $\frac{\varrho_i}{\bar{\Lambda}\kappa_i} = 1$ are used.



(a) (b)

FIG. 3: $|V_{cb}|$ extracted from $B \to D^* \ell \nu$ (a) and $B \to D \ell \nu$ (b). The dashed, solid and dotted curves correspond to $\hat{m}_b$ =5.23, 5.25 and 5.27GeV, respectively. $\bar{\Lambda} = 0.53$GeV and $\frac{\varrho_i}{\bar{\Lambda}\kappa_i} = 1$ are used.



(a) (b)

FIG. 4: $|V_{cb}|$ extracted from $B \to D^* \ell \nu$ (a) and $B \to D \ell \nu$ (b). The dashed, solid and dotted curves correspond to $\hat{m}_b$ =5.23, 5.25 and 5.27GeV, respectively. $\hat{m}_b - \hat{m}_c = 3.5$GeV and $\frac{\varrho_i}{\bar{\Lambda}\kappa_i} = 1$ are used.

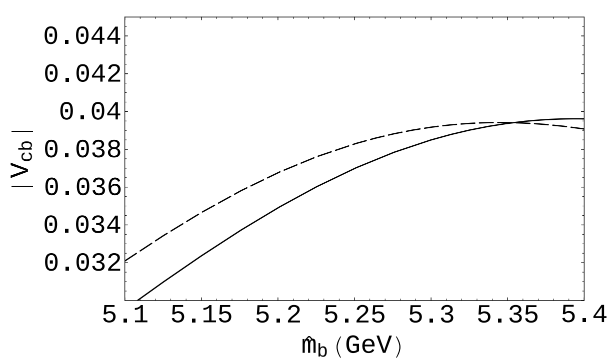


FIG. 5: $|V_{cb}|$ extracted from $B \to D^* \ell \nu$ (solid) and $B \to D \ell \nu$ (dashed) decays. $\hat{m}_b - \hat{m}_c$ =3.5GeV, $\bar{\Lambda} = 0.53$GeV and $\frac{\varrho_i}{\bar{\Lambda}\kappa_i} = 1$ are used.